The Stepped Digital Elliptic Filter

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Abstract—**The design and synthesis of various types of microwave elliptic function filters has been accomplished by a number of authors. However, one problem in this field which remains is the realization of compact narrow-band bandpass elliptic function filters. In this paper, a procedure is presented which enables this class of filters to be constructed in a compact digital form.**

Since the physical realization is in the form of an *n*-wire line, one-quarter of a wavelength long at the center frequency of the passband, where the impedance levels are stepped along the center of the coupled lines, the filter has been termed the stepped digital elliptic filter. The absence of awkward interconnections in the filter due to the stepped digital structure inherently implies that reasonable insertion loss characteristics may be achieved in the *X*-band region and above, and also simplifies the mechanical construction.

It is shown that the resonant elements in the filter, due to the design procedure adopted, are relatively insensitive to the absolute bandwidth of the filter, and consequently fractional bandwidths of approximately 30 percent and below may be readily achieved while the normalized impedance values of the elements in the network remain of the order of unity. This latter result is similar to that obtainable from conventional interdigital filters but in the case of narrow bandwidths the stepped digital filter is considerably smaller in physical size.

A systematic procedure is also formulated for the inclusion of the parasitic lumped end effect capacitances into the overall design procedure in order to maintain the equiripple passband and stopband responses.

Experimental results are presented for a five-element, 11 percent bandwidth filter and are shown to be in good agreement with theoretical predictions.

Introduction

THE KNOWN theoretical advantages which arise from designing a microwave filter to exhibit an elliptic function response in preference to a Chebyshev response have recently led to the publication of a number of contributions on this topic. These papers have in common the fact that the same electrical response is realized but they vary according to the forms of physical networks chosen to realize this elliptic response.

Saito [1] utilized the symmetrical nature of the odd-ordered elliptic filter in his realization in the form of a cascade of symmetrical two-wire coupled lines terminated in either an open- or short-circuited stub. There exist theoretical limitations upon the realizability of this structure, and the second-order case has been discussed by Matsumoto [2]. A more severe limitation which arises is due to the inability to synthesize physically realizable impedance values except in the case of bandwidths of the order of an octave.

This problem of achieving physically realizable impedance values has been common to most of the subsequent design procedures. Schiffman and Young [3] have published design tables for bandstop elliptic function filters of degree 5 using redundant unit elements. The synthesis technique used was based upon the conventional partial pole extraction procedure, thus allowing the poles of attenuation to be realized by simple and double shunt stubs. It was found that for most bandwidths practical techniques for overcoming high-impedance values had to be incorporated, but this approach fails in the case of narrow bandwidths. Levy and Whiteley [4], by using coupled-line structures, found that a systematic procedure could be formulated for the introduction of redundant unit elements into a distributed network based upon the lumped prototype, while still preserving reasonable element values. In the case of the narrow-band stopband filter, it was found that by initially resonating the low-pass prototype before incorporating the unit elements, normalized impedance values of the order of unity could be maintained. Successful narrow-band bandpass filters using this technique were not reported, however.

The design procedures described in [3] and [4] rely upon the tables published by Saal [5] for lumped low-pass prototype elliptic function filters, thus eliminating the task of constructing bounded real reflection coefficients from the prescribed insertion loss function. However, it has been shown by Horton and Wenzel [6] that the unit elements which are necessary in these realization procedures may contribute to the insertion loss characteristic, and therefore a superior characteristic may be numerically computed thus making Saal's tables inapplicable in this optimum design case.

A compact realization of wide-band elliptic function filters has also been reported by Horton and Wenzel [6], [7]. This direct interdigital realization of the basic lumped form of elliptic function filter has been termed the "digital elliptic filter." In this case no unit elements were incorporated into the network resulting in a canonic realization. This particular network possesses the fundamental advantage of direct conversion of the tabulated element values of the low-pass prototype into physical dimensions of the interdigital line. Unfortunately, this realization exhibits a number of disadvantages, the two most important being that 1) it is impossible to construct narrow-band filters and 2) difficulties are encountered when the operating range is extended into X-band and above, due to the awkward interconnection of series stubs.

Hence, it may be stated that it is difficult, if not impossible, to construct narrow-band bandpass elliptic function filters using any of the realization techniques described. A qualitative explanation may be developed, as follows, to show why these procedures are inadequate when it is required to realize narrow-band elliptic function filters.

The resonant circuits which provide the poles of attenuation in the elliptic function filter possess elements which

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are proportional to the bandwidth scaling factor and others which are inversely proportional. For small (or large) scaling factors, one set of elements become physically unrealizable if a direct digital elliptic realization is used. Similarly, in Saito's direct synthesis procedure, the parameters of the adjacently extracted two-wire lines are approximately proportional to and inversely proportional to the scaling factor, respectively. In the cases where redundant unit elements are introduced into the networks, the situation tends to deteriorate since the impedances of the unit elements are invariant to the bandwidth scaling factor, unless of course, the resonating technique of Levy and Whiteley [4] is applied in the bandstop case. However, with their technique for realizing resonant sections in cascade with a unit element, a successful solution to the narrow-band bandpass case does not necessarily follow. Similarly, other forms of resonant section realizations proposed by Levy [8] are inherently broadband.

The basic problem to be solved follows directly from this qualitative reasoning. A realization must be sought whereby the resonant sections which provide the poles of attenuation must be relatively insensitive to the bandwidth scaling factor. Such a realization is presented in this paper.

Theoretical Design Procedure

One of the two basic forms of the odd-order low-pass prototype elliptic function filter is shown in Fig. 1, and it is this cascade of pi sections which is to be realized by the stepped digital elliptic filter. Initially, we shall consider the realization of a typical section with shunt capacitances C_{r-1} and C_{r+1} and with a transmission zero at $\Omega_r = 1/\sqrt{L_r C_r}$.

The normal procedure for the conversion into a microwave bandpass filter is to apply the high-pass transformation

$$\omega \to \frac{\tan \omega_c}{\tan \omega} \tag{1}$$

where the normalized band-center frequency is at $\omega = \pi/2$ and the band-edge frequencies are situated at $\omega = \omega_c$ and $\omega = \pi - \omega_c$ in the fundamental period. However, this transformation implies that the impedance values of three of the elements are inversely proportional to $\tan \omega_c$ and one proportional to $\tan \omega_c$. For narrow bandwidths $\tan \omega_c \gg 1$; this results in at least one extreme impedance value even if the entire impedance level of the network is adjusted by means of transformer action at the input and output ports, as in case of conventional interdigital networks [9], [10].

The solution to this problem is to apply the conventional bandpass transformation

$$\omega \to a\left(\tan \omega - \frac{1}{\tan \omega}\right) \tag{2}$$

where

$$a = \frac{\tan 2\omega_c}{2}\,. \tag{3}$$

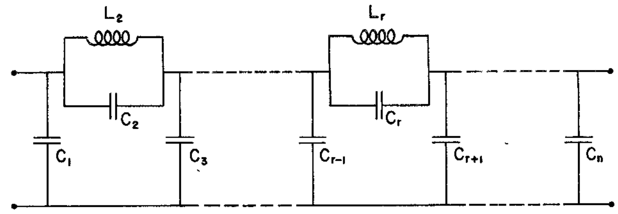

Fig. 1. The odd-order low-pass prototype elliptic function filters.

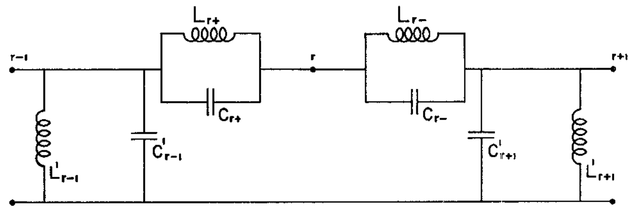

Fig. 2. A typical resonated pi section.

In this case the band-center frequency is normalized to $\omega = \pi/4$ and the band-edge frequencies $\omega = \omega_c$ and $\omega = (\pi/2) - \omega_c (\omega_c < \pi/4)$. This procedure has, of course, theoretically doubled the degree of the network but it will be shown that by using the proposed stepped digital realization the number of nonredundant coupled lines necessary to realize any given elliptic filter is equal to the degree of the low-pass prototype. Throughout this paper the degree of the network is assumed to be the degree of the resulting reflection coefficient.

Applying this resonating technique to the basic section under consideration, we generate the section shown in Fig. 2 where the pertinent element values are the following:

$$\begin{aligned} C'_{r-1} &= aC_{r-1}, \qquad C'_{r+1} = aC_{r+1} \\ L'_{r-1} &= \frac{1}{C'_{r-1}}, \qquad L'_{r+1} = \frac{1}{C'_{r+1}} \\ C_{r+} &= \frac{1}{L_{r-}} = aC_r(1 + \lambda_{r-}{}^2) \\ C_{r-} &= \frac{1}{L_{r+}} = aC_r(1 + \lambda_{r+}{}^2) \end{aligned} \tag{4}$$

where

$$\lambda_{r\pm} = \sqrt{\left(\frac{\Omega r}{2a}\right)^2 + 1} \pm \frac{\Omega r}{2a}\,. \tag{5}$$

For a narrow-band filter we have $a \gg 1$, and for most practical filter specifications this implies that $a \gg \Omega_r$ for all r. It follows immediately from (5) that λ_{r+} and λ_{r-} are of the order of unity, and hence, from (4), the admittance levels of all of the elements in the network are approximately directly proportional to the bandwidth scaling factor a. It is apparent therefore, that if the coupling into the network can be made through transformer action, then the resonant sections will become relatively invariant to bandwidth scaling. Before considering the method for achieving this transformer action, the coupled-line realization of the basic network will be discussed.

From Fig. 2, the obvious direct realization of the resonated section is in the form of a pair of 3-wire line interdigital networks connected in parallel. One network possesses lines which are short circuited to ground at one end, whereas the other is terminated in open circuits. The physical configuration is depicted in Fig. 3 where the nodes $r-1$, r, $r+1$ refer to the corresponding nodes in Fig. 2. The characteristic admittance matrices of networks 1 and 2 may therefore be identified as follows.

Network 1

$$\begin{array}{c} \begin{array}{ccc} \qquad (r-1) \qquad & \qquad (r) \qquad & \qquad (r+1) \qquad \end{array} \\ \begin{bmatrix} a[C_{r-1} + C_r(1 + \lambda_{r+}^2)] & -aC_r(1 + \lambda_{r+}^2) & 0 \\ -aC_r(1 + \lambda_{r+}^2) & aC_r[2 + \lambda_{r+}^2 + \lambda_{r-}^2] & -aC_r(1 + \lambda_{r-}^2) \\ 0 & -aC_r(1 + \lambda_{r-}^2) & a[C_{r+1} + C_r(1 + \lambda_{r-}^2)] \end{bmatrix} \end{array} \tag{6}$$

Network 2

$$\begin{array}{c} \begin{array}{ccc} \qquad (r-1) \qquad & \qquad (r) \qquad & \qquad (r+1) \qquad \end{array} \\ \begin{bmatrix} a[C_{r-1} + C_r(1 + \lambda_{r-}^2)] & -aC_r(1 + \lambda_{r-}^2) & 0 \\ -aC_r(1 + \lambda_{r-}^2) & aC_r(2 + \lambda_{r+}^2 + \lambda_{r-}^2) & -aC_r(1 + \lambda_{r+}^2) \\ 0 & -aC_r(1 + \lambda_{r+}^2) & a[C_{r+1} + C_r(1 + \lambda_{r+}^2)] \end{bmatrix} \end{array} \tag{7}$$

The interconnection of the basic sections must now be accomplished.

It may readily be shown [9], [10] that if any pair of conductors in a uniform n-wire line are at the same potential at both ends, then at any intermediate point they are at the same potential and may consequently be replaced by a single line. For the overall filter, the interconnection of consecutive sections requires that the node at the junction of the outer pair of lines of one section be at the same potential as the node at the junction of the corresponding pair of lines of the adjacent section. Since the opposite ends of the lines in network 1 are short circuited to ground, and therefore at the same zero potential, the appropriate pair of lines from adjacent sections in network 1 may be condensed into a single line [10].

Since the opposite ends of the lines in network 2 are open circuited, and therefore not necessarily at the same potential, this simple principle may not be applied directly. However, it may be shown that a sufficient condition for the condensation of open-circuited lines connected together at one end is that neither line couples directly nor indirectly to any other line which is not also open circuited. This property is of fundamental importance when consideration is given to the method of transformer coupling into the network at the input and output ports.

As an example of the line condensation procedure for a two-section (fifth-degree) filter the overall network reduces to a pair of five-wire lines as shown in Fig. 4. The characteristic admittance matrices of these two lines are

The characteristic admittance of line r to ground may be made finite by scaling the admittance level of the center rows and columns in the above matrices by a factor greater than unity, as in the case of conventional interdigital networks [9], [10], [11]. This operation does not alter the external performance of the section between nodes $r-1$ and $r+1$ if the same scaling factor is used on both networks. One additional condition which is necessary for physical realization other than the normal hyperdominancy condition, is that the distance between adjacent lines of one network must approximately equal the distance between corresponding lines on the other network, in order to obtain simple physical interconnections.

From the matrices (6) and (7) it may be seen that the admittance to ground of lines r on both networks 1 and 2 will be the same after the necessary admittance scaling. Furthermore, since the sum of the coupling element values between lines $r-1$ and r, r and $r+1$ are the same for both networks, the distance between lines $r-1$ and $r+1$ must inherently be approximately equal. In addition, since λ_{r+} and λ_{r-} are of the order of unity, the physical separation in a transverse direction of lines r on both networks must necessarily be small. These points will be discussed further after the construction of the entire network has been achieved.

We have now shown how each of the basic resonated sections in the bandpass microwave elliptic function filter may be constructed utilizing a pair of 3-wire interdigital lines.

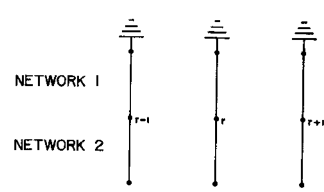

Fig. 3. The coupled-line realization of the basic resonated section.

Network 1

$$\begin{bmatrix} a[C_1 + C_2(1+\lambda_{2+}^2)] & -aC_2(1+\lambda_{2+}^2) & 0 & 0 & 0 \\ -aC_2(1+\lambda_{2+}^2) & aC_2(2+\lambda_{2+}^2+\lambda_{2-}^2) & -aC_2(1+\lambda_{2-}^2) & 0 & 0 \\ 0 & -aC_2(1+\lambda_{2-}^2) & a[C_3 + C_2(1+\lambda_{2-}^2) + C_4(1+\lambda_{4+}^2)] & -aC_4(1+\lambda_{4+}^2) & 0 \\ 0 & 0 & -aC_4(1+\lambda_{4+}^2) & aC_4(2+\lambda_{4+}^2+\lambda_{4-}^2) & -aC_4(1+\lambda_{4-}^2) \\ 0 & 0 & 0 & -aC_4(1+\lambda_{4-}^2) & a[C_5 + C_4(1+\lambda_{4-}^2)] \end{bmatrix} \tag{8}$$

Network 2

$$\begin{bmatrix} a[C_1 + C_2(1+\lambda_{2-}^2)] & -aC_2(1+\lambda_{2-}^2) & 0 & 0 & 0 \\ -aC_2(1+\lambda_{2-}^2) & aC_2(2+\lambda_{2+}^2+\lambda_{2-}^2) & -aC_2(1+\lambda_{2+}^2) & 0 & 0 \\ 0 & -aC_2(1+\lambda_{2+}^2) & a[C_3 + C_2(1+\lambda_{2+}^2) + C_4(1+\lambda_{4-}^2)] & -aC_4(1+\lambda_{4-}^2) & 0 \\ 0 & 0 & -aC_4(1+\lambda_{4-}^2) & aC_4(2+\lambda_{4+}^2+\lambda_{4-}^2) & -aC_4(1+\lambda_{4+}^2) \\ 0 & 0 & 0 & -aC_4(1+\lambda_{4+}^2) & a[C_5 + C_4(1+\lambda_{4+}^2)] \end{bmatrix} \tag{9}$$

For an elliptic function filter of degree n in the low-pass prototype version, the complete network degenerates into a pair of uniform digital n-wire lines which are one eighth of a wavelength long at the band-center frequency. Due to the shunt connection of this pair of digital networks, the complete network may be viewed as consisting of a single stepped digital line which is one quarter of a wavelength long at the band-center frequency. However, it must be remembered that the input and output terminals are situated at the centers of the first and last lines. The direct connection of these points to the external ports would yield a filter with the required electrical response, but the internal elements would be of low impedance. As mentioned previously, to overcome this problem coupling from the ports to the centers of these lines must be made using transformer action.

To accomplish this, unit elements are introduced into the network, one at each end, of unity characteristic impedance, and one eighth of a wavelength long at the band-center frequency. These unit elements may then be realized in the conventional interdigital form as shown in Fig. 5 for the case of a fifth-degree network. The number of lines in network 1 has now been increased to $n+2$ and its resulting augmented characteristic admittance matrix becomes

$$\begin{bmatrix} 1 & -1 & 0 & \cdots \\ -1 & 1 + a[C_1 + C_2(1+\lambda_{2+}^2)] & -aC_2(1+\lambda_{2+}^2) & \cdots \\ 0 & -aC_2(1+\lambda_{2+}^2) & aC_2(1+\lambda_{2+}^2+\lambda_{2-}^2) & \cdots \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{10}$$

while the characteristic admittance matrix of network 2 remains unchanged. A dual coupling system might appear to be the coupling into the open-circuited ends of the input and output lines through open-circuited coupled lines. This procedure, however, is incorrect since it changes the properties of the complete network due to the implied sufficiency condition for the line condensation of open-circuited lines. The justification for using the former method of coupling follows from the condition for line condensation of short-circuited lines [10].

The only factor which now remains to be considered is that of the admittance scaling of the entire digital line. Initially, every line in the stepped digital filter, except the lines on which the input and output ports are situated, may be scaled by the factor $1/\sqrt{a}$. The characteristic admittance matrices

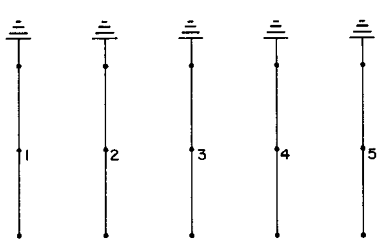

Fig. 4. Degenerate realization of a two-section filter.

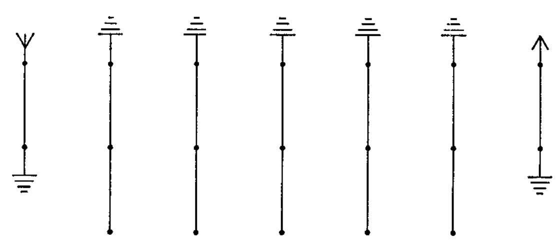
Fig. 5. Realization of fifth-order filter with transformer coupling elements.

of networks 1 and 2 then become

$$\begin{bmatrix} 1 & -\frac{1}{\sqrt{a}} & 0 & \\ -\frac{1}{\sqrt{a}} & \frac{1}{a}+C_1+C_2(1+\lambda_{2+}^2) & -C_2(1+\lambda_{2+}^2) & \\ 0 & -C_2(1+\lambda_{2+}^2) & C_2(2+\lambda_{2+}^2+\lambda_{2-}^2) & -C_2(1+\lambda_{2-}^2) \\ 0 & 0 & -C_2(1+\lambda_{2-}^2) & C_3+C_2(1+\lambda_{2-}^2)+C_4(1+\lambda_{4+}^2) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{11}$$

for network 1, and

$$\begin{bmatrix} C_1+C_2(1+\lambda_{2-}^2) & -C_2(1+\lambda_{2-}^2) & 0 \\ -C_2(1+\lambda_{2-}^2) & C_2(2+\lambda_{2+}^2+\lambda_{2-}^2) & -C_2(1+\lambda_{2+}^2) \\ 0 & -C_2(1+\lambda_{2+}^2) & C_3+C_2(1+\lambda_{2+}^2)+C_4(1+\lambda_{4-}^2) \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{12}$$

for network 2.

It follows immediately that all of the lines forming the resonant circuits are relatively insensitive to the bandwidth scaling factor since the only variation is with respect to $\lambda r\pm$ which, for narrow bandwidths, are all of the order of unity. Thus, the only significant change in the network due to a variation in bandwidth is in the values of the coupling elements at the input and output lines.

Admittance scaling may now be applied to the even-numbered lines in order to produce finite values of capacitance to ground, and to the odd-numbered lines to produce additional flexibility in the physical realization. In conventional interdigital filters, the scaling factors are normally chosen such that the capacitances to ground of all of the lines, except possibly the input and output lines, are equal. Physically, this results in bars of approximately the same width when a rectangular bar configuration is used [12]. In the present case, however, additional points must be considered.

As mentioned previously in the case of a single section, it is necessary for the complementary lines in networks 1 and 2 to be separated by approximately the same physical distance. This constraint has been shown to be inherently satisfied due to the resonated realization, and the scaling factors should therefore be chosen such that there is a minimum amount of variation in the ground and coupling capacitances throughout both networks 1 and 2. To illustrate the general procedure of admittance scaling, an example of a five-element stepped digital elliptic filter is presented.

Numerical Example

The element values for the fifth degree low-pass prototype filter to be considered may be found in [5, p. 82]. The filter with a stopband attenuation greater than 60.1 dB and a passband VSWR less than 1.22 has been chosen with an 11-percent bandwidth. The element values of this low-pass prototype are

$$C_1 = 0.9265 \qquad C_2 = 0.05866 \qquad C_3 = 1.666$$

$$C_4 = 0.1607 \qquad C_5 = 0.8363$$

with

$$\Omega_2 = 3.6119 \quad \text{and} \quad \Omega_4 = 2.3038.$$

If the center of the passband is normalized to a frequency $\pi/4 = 0.7854$ radians, then the band-edge frequency ω_c will be 0.7422 for an 11-percent bandwidth filter. From (3) we therefore have

$$a = \frac{11.54}{2} = 5.77$$

Utilizing (5) we then have

$$\lambda_{2+} = 1.361$$
$$\lambda_{2-} = 0.735$$
$$\lambda_{4+} = 1.219$$
$$\lambda_{4-} = 0.820$$

These values may now be used to determine the characteristic admittance matrices (9) and (10) for networks 2 and 1, respectively. Due to the number of algebraic operations necessary to derive these matrices, the entire procedure was programmed on a digital computer. After these matrices had been obtained, it was then necessary to apply admittance scaling to obtain a realizable network.

Initially, lines 1 and 5 on both networks were scaled by the factor $1/\sqrt{a}$. The remaining lines were scaled such that the corresponding main diagonal entries in the capacitance matrix representing network 1 were respectively equal to, and a factor 1.2 greater than, the main diagonal value of line 1 up to the center line from the input, and line 5 up to and including the center line from the output. This simple procedure appears to result in or near optimum relationship between the coupling and ground capacitances of both networks 1 and 2.

After scaling the entire matrices by the factor 7.534 in order to convert to capacitance values in a 50-ohm system, the resulting capacitance matrices for the particular numerical example under consideration were the following.

The dimensions of the bars in network 1 may be calculated from the capacitance matrix (13). However, before the dimensions of the bars in network 2 can be obtained, due account must be taken of the parasitic lumped end effect capacitances.

The following approximate compensation procedure may be adopted which neglects the interbar fringing capacitances.

Network 1

$$
\begin{array}{c}
\begin{array}{ccccccc} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{array} \\
\begin{bmatrix}
7.534 & -3.136 & 0 & 0 & 0 & 0 & 0 \\
-3.136 & 9.546 & -3.062 & 0 & 0 & 0 & 0 \\
0 & -3.062 & 11.456 & -1.395 & 0 & 0 & 0 \\
0 & 0 & -1.395 & 11.558 & -3.848 & 0 & 0 \\
0 & 0 & 0 & -3.848 & 11.558 & -3.068 & 0 \\
0 & 0 & 0 & 0 & -3.068 & 9.631 & -3.136 \\
0 & 0 & 0 & 0 & 0 & -3.136 & 7.534
\end{bmatrix}
\end{array}
\qquad (13)
$$

Network 2

$$
\begin{array}{c}
\begin{array}{ccccc} (1) & (2) & (3) & (4) & (5) \end{array} \\
\begin{bmatrix}
7.661 & -1.653 & 0 & 0 & 0 \\
-1.653 & 11.456 & -2.583 & 0 & 0 \\
0 & -2.583 & 11.269 & -2.588 & 0 \\
0 & 0 & -2.588 & 11.558 & -4.561 \\
0 & 0 & 0 & -4.561 & 9.312
\end{bmatrix}
\end{array}
\qquad (14)
$$

Using a rectangular bar configuration, Getsinger's graphs [12] may be used to calculate the physical dimensions of the filter. Using a bar thickness to ground plate spacing of 0.6, which appears to be an ideal value for all realizable bandwidths, a typical filter of the form of the sketch shown in Fig. 6 may be obtained. In this sketch, it may be noted that the capacitance to ground of the first and last lines in network 2 are partially given by the capacitance to the grounded end walls. The introduction of these grounded walls enable the transformer elements to be readily introduced into the overall physical configuration.

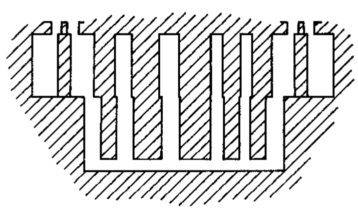

Fig. 6. Sketch of physical configuration of the designed filter.

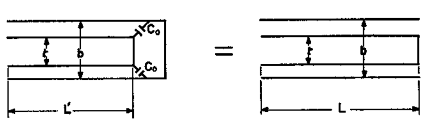

Fig. 7. Compensation for end effect capacitances in network 2.

If l is the length corresponding to an eighth-wavelength at center frequency, then the required lengh l' of network 2 will be given by

$$l' = l - 2C_0 b \qquad (15)$$

where C_0 and b are defined in Fig. 7.

This equation has been obtained from the assumption that the distribution of lumped capacitance along the end of any bar in network 2 is proportional to the distribution of capacitance to the ground plates across the width of the corresponding bar, and that the distance l-l' is considerably shorter than a quarter wavelength.

In order to maintain the correct location of the poles of infinite attenuation, due to the reduction in length of network 2, the coupling admittances in network 2 must be augmented according to the equation

$$Y_{r\pm}' = \frac{Y_{r\pm}\lambda_{r\pm}}{\tan\left(\frac{l'}{l}\tan^{-1}\lambda_{r\pm}\right)} \qquad (16)$$

where $Y_{r\pm}$ is the theoretically derived coupling admittance forming part of the resonant circuit which produces the pole of infinite attenuation at the normalized frequency $\tan^{-1}\lambda_{r\pm}$. This correction must also be applied at the center frequency to the admittances between the end lines and the grounded end walls.

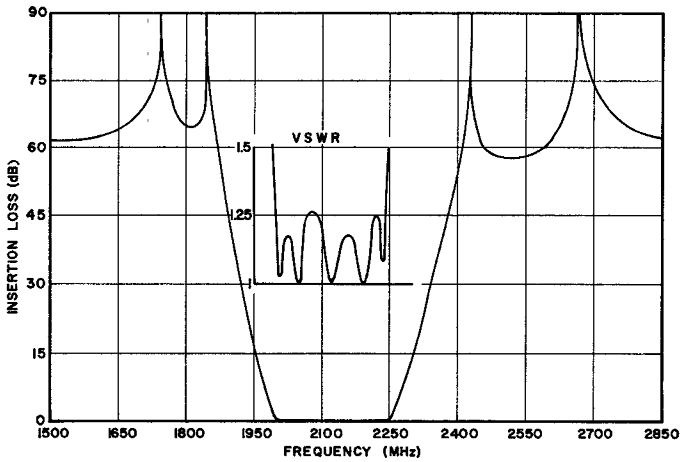

Fig. 8. Insertion loss and VSWR characteristics of experimental filter.

An experimental filter has been constructed using the values obtained from the numerical example with the above compensation for the end effect capacitances. The center frequency was 2150 MHz, from which we obtain $l=0.686$ inches. A ground plate spacing of 0.625 was chosen with a bar thickness to ground plate spacing of 0.6.

From (15) if C_0 is assumed to be the fringing capacitance for an isolated bar [12], we have

$$l' = 0.686 - 2 \times 1.2 \times 0.625 = 0.536.$$

Applying (16) to the coupling admittance given in the matrix (14) results in

$r^{\pm}$	$Y_{r\pm}$	$Y_{r\pm}'$
$0\pm$	2.400	3.41
$2+$	1.653	2.50
$2-$	2.583	3.51
$4+$	2.588	3.81
$4-$	4.561	6.27

where $Y_{0\pm}$ is assumed to be the isolated bar fringing capacitance of the end lines and is used in the calculation of the width of these lines.

The measured values of this experimental filter are shown in Fig. 8. Apart from a slight shift in the center frequency and poles of infinite attenuation, this response is in good agreement with the theoretical response.

Conclusions

In this paper a method has been presented whereby compact narrow-band bandpass elliptic function filters may be constructed. The realization is in the form of a pair of digital lines and has been called the stepped digital elliptic filter due to its basic physical configuration. Fractional bandwidths of approximately 30 percent and below may readily be obtained, but in the case of very narrow bandwidths (2 percent and below); difficulties are encountered due to the sensitivity of the response with respect to fractional percentage changes in the locations of the poles of infinite attenuation. However, due to the unique independent realization of these poles above and below the passband by the interbar coupling regions, every pole location may be individually tuned.

The main practical advantage of this filter in comparison to a conventional interdigital filter which exhibts a similar electrical response is the significant reduction in physical size. This comparison is based upon the realization of the same stopband attenuation level, the same passband VSWR and similar passband dissipation loss. The main reason for the similar dissipation loss in this reduced size filter may be attributed to the low midband group delay which in the experimental filter was less than 4 ns, resulting in a loss of 0.3 dB in the unplated brass structure.

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